Final Exam Practice Problems
Economics 140A
June 28, 2016

1. The probability mass function of X is given by:

x	1	2	3	4
$p(x)$	0.1	0.2	0.3	0.4

   (a) What is the expected value of X?
   Answer:

$$\mathbb{E}[X] = \sum xp(x)$$
$$\mathbb{E}[X] = 1 * 0.1 + 2 * 0.2 + 3 * 0.3 + 4 * 0.4$$
$$\mathbb{E}[X] = 3$$

   (b) Suppose you observe ten draws from the distribution: 2, 3, 4, 4, 4, 4, 3, 2, 1, 2. What is the sample mean?
   Answer:

$$\bar{X} = \frac{1}{n}\sum x$$
$$\bar{X} = \frac{1}{10}(2 + 3 + 4 + 4 + 4 + 4 + 3 + 2 + 1 + 2)$$
$$\bar{X} = 2.9$$

   (c) Suppose now each of the four outcomes is equally likely what is the new expected value.
   Answer:

$$\mathbb{E}[X] = \sum xp(x)$$
$$\mathbb{E}[X] = 1 * 0.25 + 2 * 0.25 + 3 * 0.25 + 4 * 0.25$$
$$\mathbb{E}[X] = 2.5$$

2. Suppose you are interested in determining the ancestry of individuals in Westeros. You are particularly interested if an individual is of Baratheon descent. You first propose the following model

$$Baratheon_i = \alpha + \beta DarkHair_i + \gamma BlueEyes_i + \delta Strength_i + \varepsilon_i$$

   Where $Baratheon_i$ is equal to 1 if an individual is of Baratheon descent and 0 otherwise, $DarkHair_i$ equals 1 if an individual has dark hair and 0 otherwise, $BlueEyes_i$ equals 1 if an individual has blue eyes and 0 otherwise, and $Strength_i$ is a measure of how much an individual can lift measured in stone.

   (a) Interpret β and δ.
   Answer: An individual with dark hair is β percentage points more to be Baratheon than an individual without dark hair, holding all else constant.

   Being able to lift one more stone increases the probability of being a Baratheon by δ percentage points, holding all else constant.

(b) Suppose you ran the above regression and obtained the following estimated regression:

$$Baratheon_i = -0.6 + 0.85DarkHair_i + 0.15BlueEyes_i + 0.03Strength_i + \varepsilon_i$$

Gendry, who does not know his father, has dark hair, blue eyes, and can lift 11 stone. What is the probability he is of Baratheon descent?

Answer: 0.73, Gendry is of Baratheon descent with 73% probability.

(c) Joffrey, who has golden hair, green eyes, and can lift only 2 stone. What is the probability he is of Baratheon descent?

Answer: -0.54, Joffrey is of Baratheon descent with -54% probability.

(d) Interpret the probability you found in part c. Does this make sense, why or why not?

Answer: The answer does not make sense, because probabilities must be between 0 and 1. Run a probit or logit instead.

3. You wish to estimate the effect of negative reviews on the number of sales of stores on Ebay. Reviews can take five values:

$$review_i = \begin{cases} 1 & \text{very bad} \\ 2 & \text{bad} \\ 3 & \text{average} \\ 4 & \text{good} \\ 5 & \text{very good} \end{cases}$$

You suggest the following model:

$$sales_i = \beta_0 + \beta_1 review_i + \beta_2 lastmonthsales_i + \varepsilon_i$$

where $sales_i$ is the number of sales in a single observed month, $review_i$ is the value of the most recent review, and $lastmonthsales_i$ is the number of sales in the previous month.

(a) Interpret β_1.

Answer: β_1 is the average change in sales associated with increasing by one review level holding last month sales constant.

(b) Do you think there is a better model? Why? If so propose a new model.

Answer: The effect of moving from very bad to bad may be different than the effect of moving from average to good. Because of this we could create binary variables for each of the possible review types.

$$sales_i = \alpha_0 + \alpha_1 verygood_i + \alpha_2 good_i + \alpha_3 bad_i + \alpha_4 verybad_i + \alpha_5 lastmonthsales_i + \nu_i$$

Now α_1 represents the average difference in sales between a very good review and an average review holding last month sales constant. α_2 represents the average difference in sales between a good review and an average review holding last month sales constant.

4. Suppose you wish to estimate the expected number of wins your favorite basketball team will have next season using payroll as the independent variable,

$$wins_i = \beta payroll_i + \varepsilon_i.$$

You believe all of our classical assumptions hold.

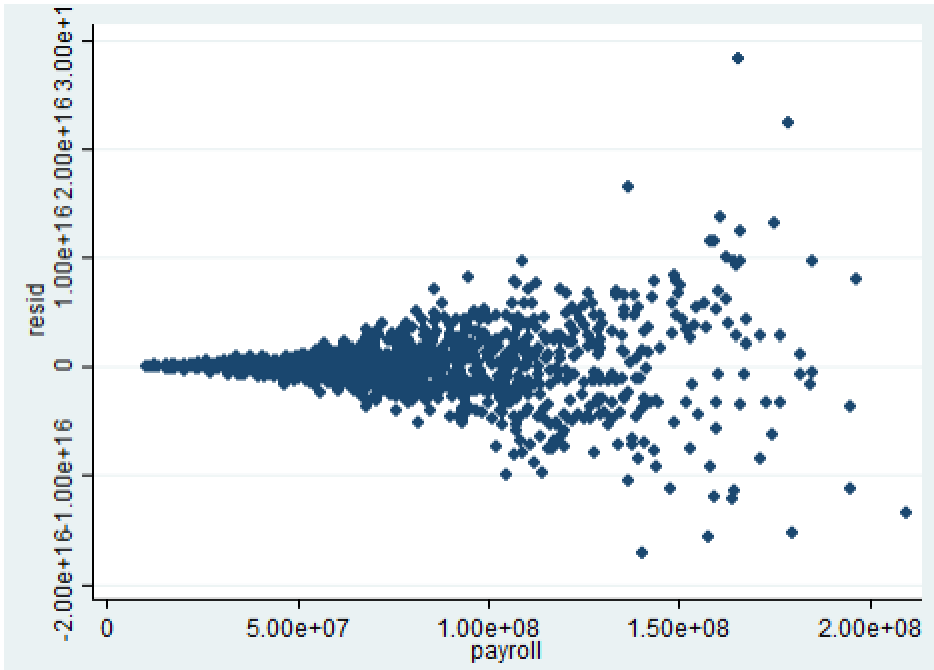
(a) Give the OLS estimator for β. Is it unbiased? State the assumptions necessary to support your claim. Is it the Best Linear Unbiased Estimator (BLUE)? State the assumptions necessary to support your claim.

Answer:

$$\hat{\beta}^{OLS} = \frac{\sum wins_i * payroll_i}{\sum payroll_i^2}$$

The OLS estimator for β is unbiased under assumptions 1-3. OLS is the best linear unbiased estimator under assumptions 1-4.

(b) After running this regression, you plot the residuals.



The above picture gives you the impression that

i. The errors are serially correlated

ii. The errors are heteroskedastic

iii. None of the above

Answer: The errors are heteroskedastic

(c) Based on your scatterplot, you do some research on the relationship between payroll and wins. Your reading leads you to believe that while all of our other classical assumptions hold, that the variance of the error term increases as the payroll increases in the following way,

$$Var(\varepsilon_i) = (p_i + p_i^2)\sigma^2$$

where p_i represents $payroll_i$.
Write down a transformed regression that corrects for the issues in (b).
Answer:

$$\begin{aligned} Var(w_i\varepsilon_i) &= \sigma^2 \\ w_i^2 Var(\varepsilon_i) &= \sigma^2 \\ w_i^2(p_i + p_i^2)\sigma^2 &= \sigma^2 \\ w_i^2 &= \frac{1}{(p_i + p_i^2)} \\ w_i &= \sqrt{\frac{1}{(p_i + p_i^2)}} \end{aligned}$$

$$weight_i * wins_i = \beta weight_i * payroll_i + weight_i * \varepsilon_i$$

$$\sqrt{\frac{1}{(p_i + p_i^2)}} * wins_i = \beta \sqrt{\frac{1}{(p_i + p_i^2)}} * payroll_i + \sqrt{\frac{1}{(p_i + p_i^2)}} * \varepsilon_i$$